

UNIVERSITY OF
ILLINOIS LIBRARY
AT URBANA-CHAMPAIGN
BOOKSTACKS



Digitized by the Internet Archive
in 2012 with funding from
University of Illinois Urbana-Champaign

#20-158

330

B385

1992:116 COPY 2

STX

The Library of the
JUL 9 1992
University of Illinois
at Urbana-Champaign

An Equilibrium Model of Quits Under Optimal Contracting

Charles M. Kahn
Department of Economics
University of Illinois

Stanley D. Longhofer
Graduate Student
Department of Economics
University of Illinois

BEBR

FACULTY WORKING PAPER NO. 92-0116

College of Commerce and Business Administration

University of Illinois at Urbana-Champaign

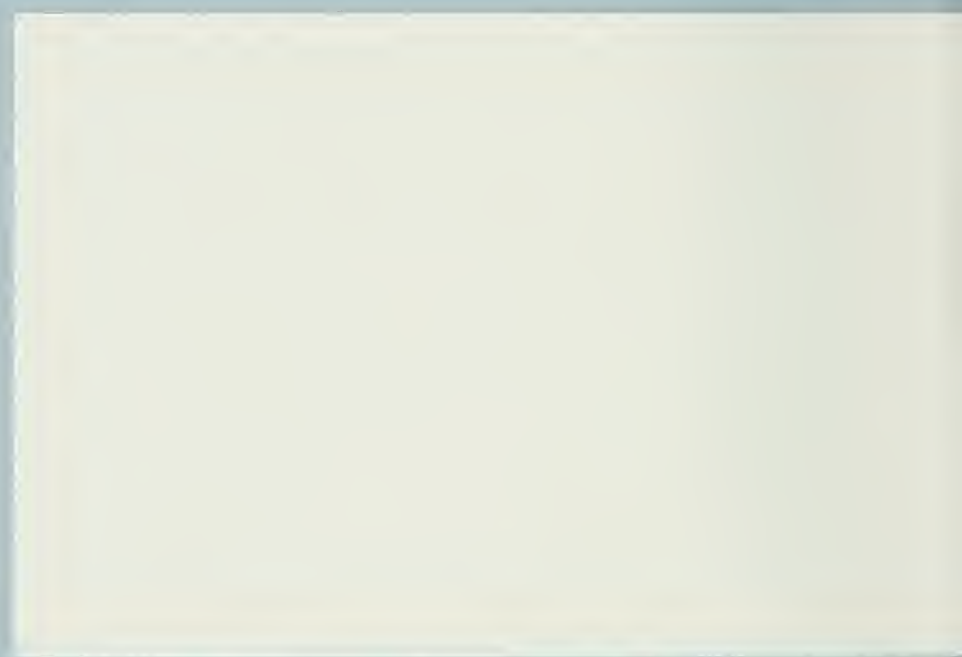
March 1992

An Equilibrium Model of Quits Under Optimal Contracting

This volume is bound without _____

#117, 123

_____ which is/are unavailable.



BEBR

FACULTY WORKING PAPER NO. 92-0116

College of Commerce and Business Administration

University of Illinois at Urbana-Champaign

March 1992

An Equilibrium Model of Quits Under Optimal Contracting

Charles M. Kahn

Stanley D. Longhofer

Department of Economics

AN EQUILIBRIUM MODEL OF QUITs UNDER OPTIMAL CONTRACTING*

Charles M. Kahn

and

Stanley D. Longhofer

Department of Economics
University of Illinois at Urbana-Champaign

Revised January, 1992

This article develops an equilibrium model of quits in a labor market and examines the effect of contracting under asymmetric information, when compared with symmetric information and noncontractual base lines. We demonstrate that quits increase in periods of high output, without postulating exogenous price rigidity.

* We are grateful to Robert Topel, Joseph Swierzbinski, Ed Prescott, the editors of this journal, two anonymous referees and participants in seminars at the Universities of Chicago, Rochester, Washington, British Columbia, California at Irvine and the National Bureau of Economic Research and the Minneapolis Federal Reserve. Funding was provided by the National Science Foundation.

More people quit their jobs when the economy is booming than when it is in a slump.¹ Most quits are voluntary changes by employees from one job to a different (presumably more lucrative) job.² If the business cycle is an across-the-board variation in the demand for labor, why should it affect mobility?

One answer appeals to fixed wage rates: If wages for current employees are sticky but wages for new hires are flexible, then we will observe increased demand for labor met by an increased mobility. Of course, we need a good explanation for the wage rigidity and why it should be less for new hires than for current employees.

This paper demonstrates that optimal contracting can generate the equivalent of wage stickiness—provided we assume imperfect information on the part of the employing firms. We construct model of contracting between workers and firms, where contracts act both to allocate labor and to reallocate risk. Aggregate implications are derived from the individual optimum contract problem faced by each firm/worker pair.³ Parties are free to make wages and other compensation as fixed or flexible as they desire.⁴

The information restriction we examine is that firms are not informed about their workers' outside opportunities. The importance of this information restriction is demonstrated through

¹ The contemporaneous correlation between quits and GNP across the business cycle is .893 (Prescott *et al.*, [1983]).

² Quits are cyclical, but unemployment among quitters is steady, and much too small to allow for significant periods of unemployment among quitters (we are grateful to Robert Topel for this information).

³ The individual optimization problem is a special case of the one described in Kahn [1985]. Alternatively, we could build an equilibrium model from the model of Ito [1984], and Ito [1986] to achieve similar results. For another example of the use of macroeconomic equilibrium models of contracting with imperfect information see Grossman, Hart, and Maskin [1983].

⁴ An alternate assumption is that, although wages are flexible, severance pay or penalties for quitting are inflexible (Ioannides and Pissarides [1983]). As we will see below, this assumption can also generate correlation between quits and output.

comparison of contracting equilibria under symmetric and asymmetric information. When the firm is fully informed about outside employment opportunities workers receive complete insurance, and the expected number of quits is independent of the business cycle. In the imperfect information contracting equilibrium, the expected number of quits is positively correlated with economy-wide output.

The organization of the paper is as follows. Section I presents the individual contracting problem. It begins with an intuitive model, showing that the contract wage schedule becomes flat in the presence of a desire for insurance against risky outside opportunities. Then we derive the characteristics of optimal contracts under more general contracting environments. Section II establishes the aggregate implications for correlation between level of output and quit rates. In Section III we relate our analysis to that of other authors and consider possible avenues for further research.

I - THE INDIVIDUAL CONTRACTING PROBLEM

The Basic Example

A worker owns an indivisible unit of labor and lives for a single period. His preferences can be described by the strictly concave expected utility function $u(y)$ where y is his income.⁵ The worker's productivity is stochastic and differs from firm to firm. Since the worker is risk-averse, he is interested in reducing his risk through a contract with a firm. All firms are risk neutral; the worker enters a relation with one before the productivity draws are realized. Let x be the worker's productivity in the firm with which he signed the contract (his "inside productivity"), and let z

⁵ As the reader will see, it is easy (and natural) to extend the model so that the worker's utility can depend on non-monetary characteristics of his job, including disutility of labor.

represent his maximum productivity at another firm (his "outside productivity"). We assume that the two are independently distributed on the interval $[a,b]$, with distributions $F(\cdot)$ and $G(\cdot)$ respectively.^{6,7} Some of the theorems below will assume that the "hazard rates" $f(x)/F(x)$ and $g(x)/G(x)$ are declining, where f and g are the density functions of F and G respectively.⁸

The contract the worker signs specifies the wage $w(x)$ he will receive as a function of inside productivity. After the productivity shock is realized, the worker can either accept the contract wage or quit and go to work at an outside firm for a wage equal to his outside productivity. Thus his income is the larger of $w(x)$ and z . If the worker quits, the firm receives zero profits; if he remains with the firm it receives $x - w(x)$. Competition among firms causes them to offer contracts which maximize the worker's expected utility subject to the firm expecting to make non-negative profits. Formally, it must choose the function $w(\cdot)$ to maximize

$$\int \int u(\max \{w(x), z\}) dF(x) dG(z) \quad (1)$$

subject to

$$\int (x - w(x))G(w(x)) dF(x) \geq 0 \quad (2)$$

Let κ be the Lagrange multiplier associated with the expected profit constraint. Then the first-order condition for the maximum is

⁶ The distribution of outside productivity G depends on the joint distribution of the worker's productivity draws at the other firms. In Section II we will discuss this distribution in more detail.

⁷ In this paper, boom periods will be ones in which the worker's observed output (that is, his output at whichever firm for which he ends up working) is high, and recessions will be periods with low observed output. This means that recessions will occur when both inside and outside productivity draws are low, while booms occur when at least one of these draws is high. Correlation between these two draws could be incorporated without seriously affecting our results.

⁸ Intuitively, the hazard rate is the probability that a worker's productivity is x given that he knows it is no higher than x . We are assuming that this probability decreases with x . The restriction to negative hazard functions is mild: $f(q)$ need not fall; it simply cannot grow exponentially.

$$u'(w(x))G(w(x)) + \kappa(x-w(x))g(w(x)) - \kappa G(w(x)) = 0 \quad (3)$$

As long as this first-order condition characterizes the outcome, it can be differentiated to yield the following lemma:⁹

LEMMA: *Contract wages increase with productivity, but less than one-for-one.*

In other words, the worker's desire for insurance leads to a flattening of the wage schedule. In this simple structure there is no way to prohibit the worker from quitting. To reduce the likelihood that he quits when his inside productivity is high, the employer shares part of the product with the worker in cases of high productivity. But the worker's desire for insurance means that the worker's contract wage does not vary one-for-one with his inside productivity; he is partially insured by having a less variable contract wage. If inside productivity rises while outside productivity remains unchanged, then quits become less likely.

This simple example provides the basic intuition as to why contract wages are "sticky"—that is, why they vary less than one-for-one with inside productivity. It also shows that expected quits are correlated with draws of inside productivity. In Section II we show that wage stickiness is part of a sufficient condition for quits to be correlated with output in the aggregate.

In this simple example it appears that the underlying problem is that employees cannot be

⁹ The flatness of the wage function depends on the first-order conditions being sufficient for the maximum, and thus can only be guaranteed if the Lagrangian is quasi-concave. Otherwise, it can happen that, while the first-order condition holds at every value of x , there is still a discontinuity in the wage function (where it jumps from one local solution to another). On the other hand, the result that wages are increasing in productivity does not depend on quasi-concavity—it holds as long as the profit function for the firm exhibits the single-crossing property.

For the more general problem examined in the next sub-section, the Appendix provides sufficient conditions for the first-order approach to yield the unique (and continuous) solution to the problem. Analogous conditions can be provided for the problem examined in this sub-section.

forced to remain with their initial firms--that ex post they will go out to other jobs if they prefer them. The next section shows that even when workers can post performance bonds, optimal contracts look much the same as they do here. The true underlying problem is not ex post enforcement but asymmetric information: Workers know their own outside opportunities better than their employers do.

Performance Bond Included

In this sub-section we derive the characteristics of the optimal contract when the contract allows a worker to post an optimal performance bond. These properties will then be used in Section II to determine when such contracts can lead to a correlation between quits and output in the aggregate.

In the situation we described in the previous sub-section wage contracts are obviously inefficient because workers are unable to commit to remain with a firm ex post. As in Harris and Holmstrom [1982], the inability to commit not to leave a firm leads to contracts with less than optimal insurance. If they could, these workers would like to arrange performance bonds to bind themselves ex ante to work for the firm. Although formal bonds are only rarely observed, many labor contracts achieve a similar effect through imperfectly vested pensions and other accrued benefits which disappear once the worker leaves the firm. Rather than develop a multi-period model which incorporates these accruals into effective bonds, we simply consider a one-period contract with explicit bonds.

If the bond were set prohibitively high, the worker would never leave the firm. Even if there were no limit to the level of bond the worker could post, a prohibitive bond would be suboptimal: It is in the firm and worker's mutual interest ex ante to stipulate that the worker will leave the firm

if his outside productivity is sufficiently high. Since only the worker knows his true outside opportunities, the bond should not be a prohibitive hurdle: it should be set high enough to allow the worker a certain level of pre-commitment, but not so high as to prohibit all mobility.

Moreover, just as the contract wage varies with the productivity of the worker, the contract penalty for quitting should vary as well. In a multi-period model where accrued benefits depend on previous experience and productivity is autocorrelated, the effective penalty for quitting does vary with current productivity. In the spirit of guaranteeing that the contract has no constraints on it other than informational constraints on outside productivity, we will consider the case where both the wage $w(x)$ and the penalty to be paid for quitting $t(x)$ are allowed to vary with inside productivity. (Of course $t(x)$ could be a net transfer from the firm to the worker; then we would call it "severance pay.") Alternatively, we could consider imposing a constant penalty t for quitting.¹⁰

Now the worker decides whether to quit based on a comparison of the pay on the job $w(x)$ and the outside pay less the penalty from quitting $z - t(x)$. Rather than working with the functions $w(x)$ and $t(x)$, it is analytically simpler and formally equivalent to work with $t(x)$ and $p(x) = w(x) + t(x)$. Observe that $p(x)$ is the difference between the pay from the firm if the worker remains with the firm and the (negative) pay he receives from the firm if he leaves; it therefore represents the *opportunity cost* of quitting under the contract. A flattened opportunity cost function (that is, one where opportunity cost rises less than one-for-one with productivity) is the analogue of wage stickiness in this environment.

In the optimal contract the functions $p(x)$ and $t(x)$ are chosen to maximize

¹⁰ If t is constant then the lemma of the previous section continues to hold—i.e. the optimal wage will be sticky. Constant t makes aggregate implications more difficult to determine. Nonetheless, some of our results continue to hold. In particular, Theorem 6 below remains valid when contracts are restricted to constant t .

$$\int \int u(\max\{p(x), z\} - t(x)) dF(x) dG(z) \quad (4)$$

subject to

$$\int [(x - p(x) + t(x)) G(p(x)) + t(x)(1 - G(p(x)))] dF(x) \geq 0 \quad (5)$$

The interpretation is as follows: Given the realization of x the worker will receive $w(x) = p(x) - t(x)$ if he stays with the firm and will pay $t(x)$ if he leaves. Thus he will leave whenever $z > p(x)$; i.e. whenever his outside wage opportunity is greater than the opportunity cost of quitting. Given x , the probability of his leaving is $1 - G(p(x))$.¹¹ If he leaves, the firm's profit is the amount $t(x)$ received from the departing worker. If he stays the firm receives x in output and pays the worker $w(x) = p(x) - t(x)$.

Again, let κ be the Lagrange multiplier for the firm's incentive constraint. The following are the first order conditions for the optimal contract:

$$\int u'(\max\{p, z\} - t) dG(z) = \kappa \quad (6)$$

$$u'(p - t) G(p) - \kappa G(p) - \kappa g(p)(p - x) = 0 \quad (7)$$

given some positive κ . For the rest of the paper we will assume that optimal contracts are characterized by these first-order conditions. This assumption is not innocuous, since the problem may be non-convex. However, a technical theorem in the appendix (Theorem A2) provides sufficient conditions for quasi-convexity, so that the optimal functions $t(x)$ and $p(x)$ are uniquely determined by equalities (6) and (7). The appendix also verifies that these sufficient conditions hold for the examples examined at the end of the next section.

¹¹ Thus, $1 - G(p(x))$ is the probability of a quit conditioned on inside productivity. Note that this is not the same as the probability of a quit conditioned on *observed* output, ex post. It is the behavior of the latter probability that determines whether quits are correlated with output, as detailed below.

THEOREM 1: *An optimal contract exists. It strictly dominates the spot market (i.e. the contract where the worker is paid his observed productivity in all states of the world). In this optimal contract, workers remain at their initial firms whenever they are most productive there. In addition they remain at their initial firms in some cases in which their productivity elsewhere is higher.*

Proof: See Appendix.

The first-order conditions given above show that, as in the pure wage contracts of the previous sub-section, optimal contracts with asymmetric information 1) offer greater inducement to more productive individuals to remain with the firm, to ensure fewer quits, and 2) provide less than perfect insurance.

As in the simpler model, the firm arranges the optimal contract so that the high productivity individual quits less often. To do so, contract wages increase with productivity at the initial firm. However, the contract wage also acts as a safety net; given a high contract wage the employee is not harmed by low draws of outside wage offers. The riskiness of outside productivity draws has less effect on a worker who has drawn a high inside productivity than on one who has drawn a low inside productivity: Conditioned on inside productivity, there is less variation in the income of a person with high inside productivity, since he is less likely to quit. The optimal contract insures across realizations of inside productivity by guaranteeing that the income in less risky outcomes lies between the extremes possible in the riskier outcomes.

These ideas can be summarized through the following theorems:

THEOREM 2: *In an optimal contract the expected frequency of quits conditioned on inside productivity, $1 - G(p(x))$, decreases as inside productivity x increases.*

Proof: This theorem follows directly from Lemma 1 and the Corollary to Lemma 4 in the Appendix. ♠

THEOREM 3: *On-the-job pay $w(x)$ and the quit penalty $t(x)$ are increasing functions of inside productivity.*

Proof: This theorem is identical to Lemma 2 in the Appendix. ♠

THEOREM 4: *The worker's total income, $\max \{p(x), z\} - t(x)$, is lowest when his productivity inside and outside the firm are both minimum. It is highest when his productivity inside is minimum but his productivity outside is maximum.*

Proof: This theorem follows as a direct consequence of Theorem 3. ♠

II - AGGREGATE FLUCTUATIONS

In this section we investigate the aggregate implications of the contracts examined so far. Our goal is to determine conditions such that optimal contracts yield pro-cyclical quits. To do so, we need a more precise specification of the distribution of outside productivity. We assume that a worker's possible jobs lie in N different sectors, in each of which his productivity is independently and identically distributed according to the same distribution function $F(\cdot)$. The worker signs with a firm in one of the sectors; if he quits he goes to work in the outside sector where his productivity (and therefore his spot pay) are highest. Thus the distribution of his outside pay if he quits is

$$G(\cdot) = F^{N-1}(\cdot)$$

(Note therefore that a declining hazard rate for F implies a declining hazard rate for G .)

One might expect that the contract described in the previous section would automatically lead

to a correlation between quits and output. Although it is easy to demonstrate based on any flattened contract that there is an inverse correlation between *inside* productivity and quits, the link between quits and ex post output (which will sometimes be output at the old firm and sometimes at the new firm) is more subtle. Let q denote an individual's observed level of output ex post. Since he will quit whenever his outside wage is greater than the opportunity cost of quitting we see that:

$$q(x,z) = \begin{cases} x & \text{if } p(x) > z \\ z & \text{if } p(x) < z \end{cases} \quad (8)$$

Figure 1 illustrates how we find the probability of a quit given observed productivity. Points in the box represent draws of (x,z) pairs. Since the worker quits whenever $z > p(x)$, points above the $p(x)$ locus represent (x,z) draws which result in a quit. Suppose we observe ex post that the worker has produced output q . Then we know that his productivity draw (x,z) must lie somewhere in the set of outcomes shown in bold. If it is in the horizontal region, then the worker quit; if it is

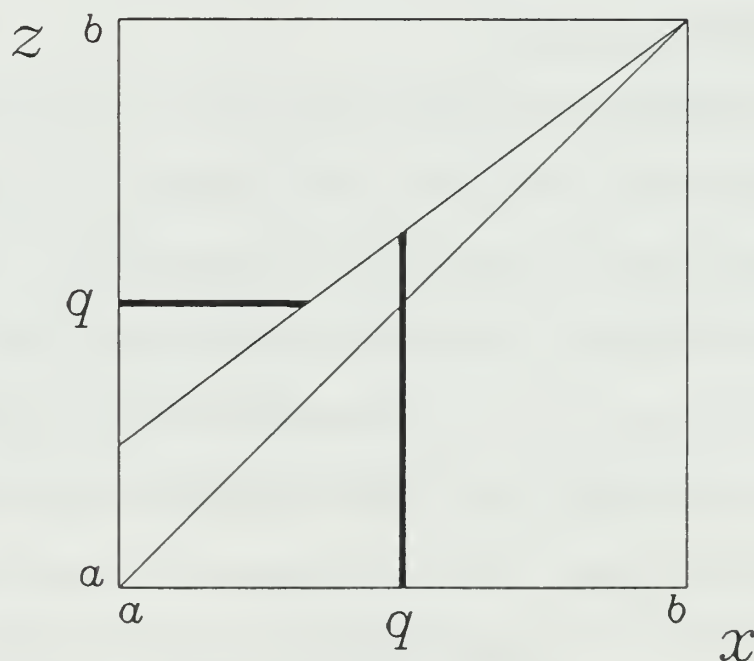


Figure 1

in the vertical portion of this set then he remained with the initial firm. Given q , the probability of a quit Q is determined by comparing the likelihood of (x,z) lying in each of these regions.

In such an environment, under symmetric information each individual would migrate to the sector in which he was most productive. If the economy allowed only spot wages, this migration would arise from each sector's offering a spot wage equal to realized productivity. If firms could offer complete ex ante insurance contracts, migration would still be efficient: the contracts would be written to induce the worker to take the most productive offer ex post.¹² By the *iid* assumption $(N-1)/N$ of the time the most productive offer, ex post, is with a firm other than the initial contracting firm. Thus, in this economy we would observe quits $(N-1)/N$ of the time independent of observed levels of productivity in the economy. Since workers are randomly paired with firms ex ante (there is no way to know with which firm they are most likely to be productive), there is no relationship between their realized productivity and the probability of a quit.

Contrast this with the effects of the private information contracts we have investigated. Is there positive correlation between quits and output? It turns out that under the optimal contract, quits increase with output whenever the opportunity cost of quitting increases less than one-for-one with inside productivity. The following theorem gives a sufficient condition for this probability to be increasing in q —i.e. that the aggregate level of quits is correlated with the aggregate level of output in the economy.

THEOREM 5: *Suppose F has a declining hazard function (i.e. $f(x)/F(x)$ is a decreasing function of x). If $p'(x) < 1$ in the optimal contract, then the expected number of quits increases with observed*

¹² Specifically, the firm will guarantee the worker his expected wage $\bar{w}$. If the worker's outside productivity is higher than $\bar{w}$ the worker will quit, receive z from the new firm, and pay the original firm $z - \bar{w}$. Note that this is possible under symmetric information since the firm can verify the worker's outside productivity.

*level of output.*¹³

Essentially, this theorem requires that the optimal contract be "sticky"—i.e. that $p(x)$ increases slowly relative to x . The proof of this theorem shows that this causes $F(q)/F(p(q))$ to increase. Intuitively, it can be shown that this expression is the probability of a quit conditional on the information that outside productivity is q and that inside productivity is less than q —that is, when outside productivity is q and a quit would be efficient. In other words, the theorem requires that contracts be structured such that the rate of inefficient retentions declines as outside productivity increases.

We conclude this section with two examples where contracts satisfy the conditions we have outlined. Calculations are relegated to the appendix.

THEOREM 6: *If F is a uniform distribution on $[a,b]$ then an optimal contract satisfies the first order conditions (6) and (7). Furthermore, quits and output are positively correlated.*

This result holds for all concave utility functions and can easily be extended to cases of heterogeneous populations. The proof of this theorem also holds if we restrict attention to contracts where the penalty for quitting is constant.

¹³ Note that this theorem is really more general than it appears at first glance. In particular, even if contracts are not formed as described in the previous section, there will be a correlation between quits and output as long as F has a declining hazard function and $p(x)$ and $F(x)/F(p(x))$ are increasing functions of x . Under the optimal contract described in the previous section, this simplifies to the hypothesis given in the theorem. Extensions of this result are examined in Kahn [1987].

THEOREM 7: *Suppose the worker has constant absolute risk aversion r . If the distribution function F satisfies*

$$\frac{d}{dx} \ln(f(x)/F(x)) > -r \exp\{-r(b-a)\}$$

for all x in $[a,b]$, then an optimal contract satisfies the first order conditions (6) and (7), and has $p'(x) < 1$.

In other words if the hazard function does not decrease too quickly throughout its range, the contract is sticky, with opportunity cost increasing less than one-for-one with productivity. Recall that Theorem 5 states that if the hazard function declines (at all), a sticky opportunity cost function implies pro-cyclical quits. We can combine these two theorems:

COROLLARY:¹⁴ *If individuals have constant absolute risk aversion r , then there exists a positive number L such that if $F(\cdot)$ satisfies*

$$0 > \frac{d}{dx} \ln\left(\frac{f(x)}{F(x)}\right) > -L$$

quits are pro-cyclical.

III - SUMMARY AND CONCLUSIONS

We have shown that in an economy with optimal employment contracting but imperfect information about employees' outside offers there can be a positive correlation between aggregate output and the quit rate, where this correlation cannot naturally be obtained under symmetric information with flexible contracts. The following is an intuitive interpretation of this result. In this economy, compensation performs two functions: First, it is part of an insurance scheme provided

¹⁴ Given L it is easy to generate distributions which satisfy the condition of this corollary. Its most restrictive property is that it must have an atom at the bottom of the distribution. This poses no problem for any of the theorems of this paper, which continue to hold with such an atom. Moreover, all the aggregate results will continue to hold if that atom is smoothed out.

by the contracting firm, and second, it serves as a signal directing the employee to the most productive job available. These two functions conflict; the optimal contract is a compromise. Under the optimal contract, workers tend to move on to higher productivity jobs should those jobs become available, but they do not move as often as they would under perfect information. Income variability is moderated by the contract, but not as much as it would be with symmetric information insurance.

As a result, we find in this economy some of the effects of a fixed-price contract. The wage schedule is an increasing function of productivity in an optimal contract; however, the opportunity cost of quitting increases less than one-for-one with inside productivity. To see this, consider the extreme cases. When inside productivity is at the maximum, it is sub-optimal for the employee to quit, since outside productivity cannot be greater. The optimal contract ensures that he does not quit. In contrast, when inside productivity is at a minimum, the employee would always be more productive elsewhere. Nonetheless the optimal contract does not lead him to quit always. Outside spot wages entail a risk; for partial insurance against the risk the contract offers to retain the employee with fixed compensation in some situations of low outside productivity, even though it means the employee will remain with the firm when he would be more productive elsewhere.

Ceteris paribus, "good times" mean increases in both inside and outside productivity. But when outside productivity increases, spot wages increase one-for-one. When inside productivity increases, the opportunity cost of quitting (in the simplest contract, the wage) increases less than one-for-one. The result is an increase in quits, on average, when productivity is high.

The major limitations in this analysis stem from the restrictions on the functional forms we have used. We have limited ourselves to risk-neutral firms. A more desirable formulation would allow the possibility that owners are risk averse (as, for example in Grossman and Hart [1983]) or, equivalently, that liability is limited (as Kahn and Scheinkman [1984]). The form of distribution we

have considered for productivity shocks is extremely restrictive. It would be worthwhile both to allow more general functional forms for individual shocks (as in Kahn [1987]) and to drop the assumption of independence across individuals, allowing for correlated aggregate fluctuations. An additional desirable extension would handle production functions with diminishing returns to employment and multiple periods, as in Ioannides and Pissarides [1983].

The other set of extensions worth considering would vary the set of permissible contracts. For instance, in contrast to the article by Grossman, Hart, and Maskin, our analysis does not assume aggregate variables are observable in sufficient time to make contracts depend on them. If this possibility were included, then a natural extension would be to link real fluctuations in quits and output with nominal shocks, as in Ball and Romer [1990].

We have structured this paper so that informational imperfections provide the only limitations on the power of parties to contract. Many authors have emphasized other limitations to the power to contract. Harris and Holmstrom [1982] emphasize restrictions on employee's ability to make commitments. More recent work has examined the effects of both parties' inability to commit not to renegotiate the contract (for a recent survey see Tirole [1990]).

In contrast to Prescott and Townsend [1982], we have not included randomized strategies in the contract. In cases where the conditions for quasi-convexity do not hold, there will in general be gains to the parties from the inclusion of such randomizations. (For an examination of these possibilities see Tsoulouhas [1991].)

Thus many modifications to the set of permitted contracts would be of interest. Nonetheless, we predict that none of these extensions will reverse our basic result, since none of them will in general lead to symmetric information allocational efficiency.

APPENDIX

Throughout this appendix we will assume that F is distributed on the interval $[a, b]$ and has a positive and continuous density function. The proofs also hold if the distribution has an atom at the left end point.

We will consider a slightly generalized version of the problem in the text:

PROBLEM A: Find measurable functions $p(\cdot)$ and $t(\cdot)$ to maximize

$$\iint u(\max\{p(x), z\} - t(x)) dF(x) dG(z) \quad (A1)$$

subject to

$$\int (x - p(x) + t(x)) G(p(x)) + t(x)(1 - G(p(x))) dF(x) \geq h \quad (A2)$$

for some constant h .

We can regard h as the firm's minimum profit level in some social welfare function.

Proof of Theorem 1:

We will demonstrate through the use of several lemmas that without loss of generality $p(\cdot)$ and $t(\cdot)$ can be restricted to a bounded set of non-decreasing functions. Since such a set is compact, existence follows along lines of the demonstration of Kahn [1992]. After proving existence we will turn to the final two claims of the theorem.

Without loss of generality we can bound the function $p(\cdot)$ as follows:

$$a \leq p(x) \leq b \quad (A3)$$

If $p(x) < a$ in some otherwise feasible contract, then $G(p(x))$ equals zero, and the identical outcome

can be attained by setting $p(x) = a$. If $p(x) > b$ then $G(p(x)) = 1$ and the identical outcome can be attained by decreasing $p(x)$ to b and decreasing $t(x)$ by the same amount.

Furthermore, it is clear that in any optimum, constraint (A3) will be binding. Let us define

$$\omega(p, t) = \int u(\max\{p, z\} - t) dG(z)$$

$$\pi(x, p, t) = t + G(p)(x - p)$$

Clearly, $\omega(p(x), t(x))$ is the worker's expected utility, given his opportunity cost and penalty functions, and $\pi(x, p(x), t(x))$ is the firm's profit given these same functions and the worker's observed inside productivity. With these definitions we can simplify the problem as follows:

PROBLEM B: Find $\{t(\cdot), p(\cdot)\}$ to maximize

$$\int \omega(p(x), t(x)) dF(x) \tag{B1}$$

subject to

$$\int \pi(x, p(x), t(x)) dF(x) = h \tag{B2}$$

$$a \leq p(x) \leq b \tag{B3}$$

LEMMA 1: In the optimal contract $p(x)$ must be a weakly monotone increasing function.

Proof: The proof follows a variational argument; if p decreased between two points, then by switching values of p (and t) around those points, we could increase expected profit without affecting utility. ♠

For each $p(\cdot)$ for which (B3) holds—and in particular for the optimal $p(\cdot)$ —we can consider

the sub-problem:

PROBLEM C: Find $t(\cdot)$ to maximize (B1) subject to (B2).

Problem C is a convex problem with a unique solution for each p . The first order conditions of this problem imply that if t_1 is the solution when $p(x) = p_1$ and t_2 is the solution when $p(x) = p_2$, then

$$\int u'(\max\{p_1, z\} - t_1) dG(z) = \int u'(\max\{p_2, z\} - t_2) dG(z) \quad (B4)$$

LEMMA 2: In any contract solving C (and in particular in a solution to B), $p - t$ increases with p almost everywhere (i.e. w is increasing in p). Also, t increases with p a.e.

Proof: Without loss of generality let $a < p_1 < p_2 < b$. Suppose $t_1 \geq t_2$. Then

$$\max\{p_1, z\} - t_1 < \max\{p_2, z\} - t_2 \quad \forall z < p_2$$

contradicting (B4). Similarly, suppose $p_1 - t_1 \geq p_2 - t_2$. Then

$$\max\{0, z - p_1\} + (p_1 - t_1) > \max\{0, z - p_2\} + (p_2 - t_2) \quad \forall z > p_1$$

contradicting (B4). ♠

We are now in a position to show that t is bounded, from which existence follows directly.

LEMMA 3: In an optimal contract

$$h + 2a - b < t(x) < h + 2b - a \quad (B5)$$

Proof: From Lemma 2, $|t_2 - t_1| < b - a$ for any t_1 and t_2 in a contract solving Problem C. But the minimum t must be greater than $a + h$ —otherwise expected profits are negative. Similarly the maximum t must be less than $b + h$. Thus (B5) holds given any $p(\cdot)$ and in particular for the

optimal $p(\cdot)$. ♣

Having placed bounds on $t(\cdot)$ and $p(\cdot)$, we know that the candidates for the optimum form a compact set (see Kahn [1992]). Since by Lemmas 1 and 2 they are weakly monotone, existence follows immediately. This proves the first claim in Theorem 1. ✓

LEMMA 4: *In the optimal contract $p(x) > x$ almost everywhere, so that workers never quit if $x \geq z$ but do not always quit if $x < z$.*

Proof: Take a feasible contract $\{p(\cdot), t(\cdot)\}$ and a particular value x . Let $\pi = \pi(x, p(x), t(x))$. Consider replacing $p(x)$ by some new value p , while adjusting t so as to leave profit in state x unchanged:

$$t = \pi - (x-p) G(p)$$

Assume u is differentiable (if not, simply use its right derivative in the following formulas). The effect on expected utility of a change in p is:

$$\begin{aligned} \frac{d\omega}{dp} &= \frac{\partial \omega}{\partial p} + \frac{\partial \omega}{\partial t} \frac{dt}{dp} \\ &= G(p) u'(p-t) - \left[G(p) u'(p-t) + \int_p^b u'(z-t) dG(z) \right] (G(p) + (p-x)g(p)) \\ &= G(p) \int_p^b (u'(p-t) - u'(z-t)) dG(z) - (p-x)g(p) \int_a^b u'(\max\{p, z\} - t) dG(z) \end{aligned}$$

The first term is always positive by the concavity of the utility function, and the second term is non-negative when $p \leq x$. Thus, $d\omega/dp$ is positive if $p < x$ or if $p = x$ and $a < x < b$. ♣

COROLLARY: *The opportunity cost function p is increasing somewhere (more precisely it is not the case that p is constant a.e.).*

Proof: Lemma 1 shows that p is non-decreasing a.e.. To show that it is increasing somewhere, note that $x < p(x) < b$ on (a,b) . Therefore, there is no value c such that $p(x) = c$ a.e. on (a,b) . ♠

Lemma 4 states that increasing p increases utility conditional on x without changing profits as long as p is not greater than x . Therefore if $p(x) \leq x$ for a non-negligible set of values x , then the initial contract cannot be optimal, which proves the final claim of Theorem 1. ✓

Finally, note that the behavior in a spot market is identical to a "contract" in which $p(x) = x$ and $t(x) = 0$. Applying the previous paragraph's result, such a contract is not optimal, proving the second part of Theorem 1. ♠

Proof of Theorem 5:

Using the definition given by (8) in the text, the probability of a quit conditional on $q(x,z) = q$ is

$$Q(q) = \frac{g(q)F(p^{-1}(q))}{f(q)G(q) + g(q)F(p^{-1}(q))} = \frac{P}{1 + P}$$

where

$$P = \frac{F(p^{-1}(q))g(q)}{f(q)G(p(q))} = (N-1) \left[\frac{F(p^{-1}(q))}{F(q)} \right] \left[\frac{F(q)}{F(p(q))} \right]^{N-1}$$

Now, the second bracketed expression is increasing in q since

$$\text{sgn } \frac{d}{dq} \frac{F(q)}{F(p(q))} = \text{sgn } \left[\frac{f(q)}{F(q)} - \frac{f(p(q))}{F(p(q))} p'(q) \right] > 0$$

whenever $p(q) > q$ and $p'(q) < 1$ (recall F has a declining hazard function). Likewise, the first bracketed expression is increasing in q since

$$\text{sgn } \frac{d}{dq} \frac{F(p^{-1}(q))}{F(q)} = \text{sgn } \left[\frac{f(p^{-1}(q))}{F(p^{-1}(q))} p^{-1'}(q) - \frac{f(q)}{F(q)} \right] > 0$$

Thus the probability of a quit for a single individual is correlated with his own output. ♠

Derivation of Sufficient Conditions for the First-Order Condition Approach to be Valid:

While Theorem 1 shows that an optimal contract exists, since the problem may be non-convex it does not guarantee that the first order conditions (6) and (7) characterize the solution. Our goal then is to find sufficient conditions for this to happen. To do this we reformulate the problem with a Lagrange multiplier. Since the problem is not convex, there is no guarantee that the reformulation is equivalent. However, we can establish:

THEOREM A1: $\{t(\cdot), p(\cdot)\}$ is an optimum contract if (a) there exists a number κ such that for almost all $x \in [a, b]$, $\{t(x), p(x)\}$ is the unique solution to equations (6) and (7) of the text, and (b) $\int \pi(x, p(x), t(x)) dF(x) = 0$.

Proof: The contract is optimal if (b) is satisfied and if for almost all x , $\{t(x), p(x)\}$ maximizes

$$\omega(p, t) + \kappa \pi(x, p, t) \tag{D1}$$

Equations (6) and (7) are the first order conditions for maximizing (D1). For almost all x the maximum to (D1) is interior so a unique solution to the first order conditions is sufficient for an optimum. ♠

Thus, we must find sufficient conditions for the existence of κ and a unique solution to equalities (6) and (7) of the text. Recall that we use the variable $w = p - t$ to denote the pay received by the worker who remains with the firm. Given p and κ , the following equation (a transformation of equation (6) in the text) implicitly defines $w(p, \kappa)$:

$$\int_a^b u'(w + \max\{0, z - p\}) dG(z) = \kappa \quad (D2)$$

We wish to use (7) in the text to define $x(p)$, the inverse of the optimal schedule $p(x)$. The following technical theorem states sufficient conditions on preferences and distributions for the first order approach to be valid:

THEOREM A2: *If the function*

$$\Phi(p) = p - [u'(w(p, \kappa)) - \kappa] \frac{G(p)/g(p)}{\kappa} \quad (D3)$$

is strictly monotonic on $[a, b]$ for all κ in $[u'^{-1}(h+2b-a), u'^{-1}(h+a)]$, then there exists a κ such that in the optimal contract

$$p(x) = \Phi^{-1}(x) \quad (D4)$$

$$t(x) = p(x) - w(p(x), \kappa) \quad (D5)$$

Proof: Φ is derived from (7); when it is monotonic there is a unique solution to (6) and (7). Expected profits are continuous in κ ; they are negative for the minimum κ in the indicated interval and positive for the maximum κ . Thus for some intermediate value of κ , the conditions of Theorem A1 are satisfied. ♠

Proof of Theorems 6 and 7:

Here we will apply conditions (D3)-(D5) to specific cases, generating conditions in which

increased output correlates with increased quits.

THEOREM 6: *If $F(x) = x/(b-a)$ (so that F is uniform on $[a,b]$) then Φ is strictly monotonic so the optimal contract is characterized by the first order conditions. In this contract quits are correlated with output.*

Proof: First note that $f(x)/F(x) = 1/(x-a)$, so F has a declining hazard function. Rewriting (D3) we get

$$\Phi(p) = p + \frac{G(p)}{g(p)} + \frac{-u'(w(p))}{\kappa} \frac{G(p)}{g(p)}$$

Each of these terms is increasing in p (note that since u is concave, $-u$ is convex). Thus, by Theorem A2 we can use the first order conditions. To show correlation between quits and output we calculate the probability of a quit given ex post output and take the first derivative. As in the proof of Theorem 5, we have

$$Q(q) = \frac{g(q)F(p^{-1}(q))}{f(q)G(q) + g(q)F(p^{-1}(q))}$$

Substitution gives us:

$$\begin{aligned} Q(q) &= \frac{\frac{N-1}{b-a} \left(\frac{q-a}{b-a} \right)^{N-2} \frac{1}{b-a} \left(\frac{qN-a}{N-1} - \frac{(q-a)u'(q-t)}{\kappa(N-1)} \right)}{\frac{1}{b-a} \left(\frac{q-a}{b-a} \right)^{N-1} + \frac{N-1}{b-a} \left(\frac{q-a}{b-1} \right)^{N-2} \frac{1}{b-a} \left(\frac{qN-a}{N-1} - \frac{(q-a)u'(q-t)}{\kappa(N-1)} \right)} \\ &= \frac{\frac{\kappa(qN-a)}{q-a} - u'(q-t)}{\kappa + \frac{\kappa(qN-a)}{q-a} - u'(q-t)} \end{aligned}$$

The derivative of Q has the same sign as the derivative of the numerator:

$$\frac{\kappa(qN-a)}{q-a} - u'(q-t)$$

The derivative of the first term with respect to q is positive. Thus, $\partial Q(q)/\partial q$ is positive if

$$\frac{d u'(q-t)}{dq} < 0$$

Noting that $q-t = w(p^{-1}(q))$ and recalling that $w'(\cdot) > 0$ (Lemma 2) completes the proof. ♠

Note that in the case where t is restricted to be constant, this proof is unchanged.

THEOREM 7: *If individuals have utility functions $u(w) = -\exp\{-rw\}$ and*

$$\frac{d}{dx} \ln[f(x)/F(x)] > -r \exp\{-r(b-a)\}$$

Then the optimal contract is characterized by the first order conditions. Furthermore, in this contract $p - x$ is decreasing (i.e. $p'(x) < 1$).

Proof: We will show that $\frac{d}{dp}(p - \Phi(p)) < 0$, implying that Φ is strictly monotone, and thus that the first order approach is valid. Moreover, it implies that $p'(x) < 1$ (recall that $\Phi(p) = x$). Analytically it is easier to consider the natural log of $p - \Phi(p)$ (since this is a monotonic transformation it will not affect the sign of the derivative). Taking this derivative gives us:

$$\begin{aligned} \frac{d}{dp} \ln[p - \Phi(p)] &= \frac{d}{dp} \ln[u'(w(p, \kappa)) - \kappa] - \frac{d}{dp} \ln\left[\frac{g(p)}{G(p)}\right] - \frac{d}{dp} \ln \kappa \\ &= \left[\frac{u''(w(p, \kappa))}{u'(w(p, \kappa)) - \kappa} \right] \frac{d}{dp} w(p, \kappa) - \frac{d}{dp} \ln\left[\frac{(N-1)f(p)}{F(p)}\right] \end{aligned} \quad (E1)$$

Simplifying condition (D2) above we see:

$$\kappa = u'(w(p, \kappa)) G(p) + \int_p^b u'(w+z-p) dG(z) \quad (E2)$$

This implies:

$$\frac{d}{dp} \ln [p - \Phi(p)] = \frac{-r u'(w(p, \kappa))}{u'(w(p, \kappa))(1 - G(p)) - \int_p^b u'(w+z-p) dG(z)} \frac{d}{dp} w(p, \kappa) - \frac{d}{dp} \ln \left[\frac{f(p)}{F(p)} \right] \quad (E3)$$

In addition, if we totally differentiate (E2) we see:

$$\frac{d}{dp} w(p, \kappa) = \frac{\int_p^b u'(w+z-p) dG(z)}{\kappa} \quad (E4)$$

Substituting this back into (E3) we get:

$$\begin{aligned} \frac{d}{dp} \ln [p - \Phi(p)] &= \frac{-r u'(w(p, \kappa))}{u'(w(p, \kappa))(1 - G(p)) - \int_p^b u'(w+z-p) dG} \frac{\int_p^b u'(w+z-p) dG}{\kappa} - \frac{d}{dp} \ln \left[\frac{f(p)}{F(p)} \right] \\ &\leq \frac{-r u'(w(p, \kappa)) \int_p^b u'(w+z-p) dG(z)}{u'(w(p, \kappa)) (1 - G(p)) \kappa} - \frac{d}{dp} \ln \left[\frac{f(p)}{F(p)} \right] \\ &< \frac{-r \int_p^b u'(w+z-p) dG(z)}{(1 - G(p)) \kappa} + r \exp\{-r(b-a)\} \\ &\leq \frac{-r u'(w(a)+b-a)}{u'(w(a))} + r \exp\{-r(b-a)\} \\ &= 0 \end{aligned} \quad (E5)$$

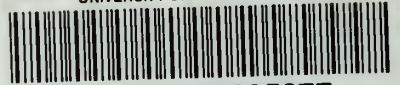
References

- Ball, Laurence and David Romer, (1990) "Real Rigidities and the Non-Neutrality of Money," *Review of Economic Studies*, vol 57, pp 183-203.
- Grossman, Sanford J. and Oliver D. Hart, (1983) "Implicit Contracts Under Asymmetric Information," *Quarterly Journal of Economics*, 98 (supplement), pp. 123-56.
- _____, _____ and Eric S. Maskin, (1983) "Unemployment with Observable Aggregate Shocks," *Journal of Political Economy*, 91 (6), pp. 907-28.
- Harris, Milton and Bengt Holmstrom, (1982) "A Theory of Wage Dynamics," *Review of Economic Studies*, 49, pp. 315-33.
- Ioannides, Yannis M. and Christopher A. Pissarides, (1983) "Wages and Employment with Firm-Specific Seniority," *Bell Journal of Economics*, pp 573-580.
- Ito, Takatoshi, (1984) "Implicit Contracts with Costly Search: The Incentive constraint in Severance Payments," University of Minnesota working paper.
- _____, (1986) "Implicit Contracts and Risk Aversion," in Heller, Walter P., Ross M. Starr and David A Starett, eds., *Equilibrium Analysis: Essays in Honor of Kenneth J. Arrow, Volume II*, Cambridge University Press, pp. 265-287.
- Jovanovic, Boyan, (1984) "Job Matching and the Theory of Turnover," *Journal of Political Economy*, 87 (5), 1984, pp. 972-89.
- Kahn, Charles M., (1985) "Optimal Severance Pay and Incomplete Information," *Journal of Political Economy*, vol. 93, pp 435-451.
- _____, (1987) "On the Correlation of Quits and Output," working paper, University of Chicago.
- _____, (1992) "Existence and Characterization of Optimal Contracts on a Continuous State Space," forthcoming *Journal of Economic Theory*.
- _____, and Jose A. Scheinkman, (1985) "Optimal Employment Contracts with Bankruptcy," *Journal of Economic Theory*, Vol 35 pp 343-365.
- Prescott, Edward C. and Robert M. Townsend, (1984) "Pareto Optima and Competitive Equilibria with Adverse Selection and Moral Hazard," *Econometrica*, vol 52, No 1. pp 21-45.
- Prescott, Edward C., Anne Guenther, Patrick Kehoe and Rudolfo Manuelli, (1983) "Can the Cycle be Reconciled with a Consistent Theory of Expectations," Federal Reserve Bank of Minneapolis Research Department working paper.

Tirole, Jean, (1990) "Collusion and the Theory of Organizations," working paper, Massachusetts Institute of Technology.

Tsoulouhas, Theofanis, (1991) "Tournaments as Pure Randomization Devices in Labor Contracts," unpublished manuscript, University of Illinois.

UNIVERSITY OF ILLINOIS-URBANA



3 0112 060295877